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VAGUE BINARY SOFT SETS AND THEIR PROPERTIES

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Abstract: The aim of this paper is to introduce the novel concept vague binary soft sets and to characterize some of its properties.
Keywords: Vague binary set, vague binary soft set, vague binary soft equal set, vague binary soft complement, vague binary soft AND operation, vague binary soft OR operation
Notations: $\ddot{F}$(e) represents the e-approximate element under the mapping $\ddot{F}$. Usual set theoretical operations with double dot on top will be used for vague binary soft set operations. Vague binary soft set is denoted by VBSS. Collection of all vague binary soft sets over the common universe U_1, U_2 under fixed parameter set A is denoted by VBSS$(U_1, U_2)_A$

1. Introduction

In most of the real life situations, humanity is bound to face with loss of data, unclear data, game of chance etc. To overcome such situations, classical probability theory (Gerolamo Cardano, 1501-1575)[5], fuzzy set theory (Zadeh, 1965)[2,5], rough set theory (Pawlak, 1982)[2,5], intuitionistic fuzzy set theory (Attanassov, 1986)[2,5],vague set theory (Gaue & Buehrer, 1993)[5], theory of interval mathematics (Moore, 1996)[5], neutrosophic set theory (Smarandache, 2005)[2] etc have played an important role. Inspired from 'Pawlak's work done in 1993', Molodtsov introduced soft sets in 1999. It has loosened all the existing rigid structure of classical sets by providing plenty of parameterization tools. Hybrid structures like fuzzy soft [3], soft fuzzy [3], neutrosophic soft [2], vague soft [2,4] etc developed later to make things more easier. All of them found rich with parameterization tools.

Later in 2016, Ahu Acikg$\ddot{o}$z [1] introduced binary soft sets with its operations and concluded that soft set can be given on n-dimension initial universal sets with a parameter set like F: A $\longrightarrow \prod_{i=1}^{n} P(U_i)$, where $U_i$are initial universal sets for 1⩽i⩽n and A is the parameter set. Vague binary sets and Vague binary soft sets with two initial universal sets are introduced. Some of the basic operations for vague binary soft sets like union, intersection, complement, AND operation, OR operation, Cartesian product are introduced in this paper. Terms like null vague binary soft set, absolute vague binary soft set are also introduced.

2.Preliminaries

Definition 2.1:[2,5]

A vague set A in the universe of discourse U={u_1, u_2, …….,u_n} is characterized by two membership functions given by
(1) a truth membership function t_A: U ⟶ [0,1]
(2) a false membership function f_A: U⟶ [0,1] where $t_A(u_i)$ is a lower bound of the grade of membership of $u_i$derived from the " evidence for u_i" and $f_A(u_i)$ is a lower bound on the negation

of u derived from the "evidence against u_i" and $t_A(u_i)$+$f_A(u_i)$⩽1. Thus the grade of membership of u_i in the vague set A is bounded by a sub-interval [$t_A(u_i)$, 1−$f_A(u_i)$] of [0,1]. This indicates that if the actual grade of membership is $\mu(u_i)$ then $t_A(u_i)\leqslant\mu(u_i)\leqslant f_A(u_i)$. The vague set A is written as A ={(u_i,[$t_A(u_i)$,1−$f_A(u_i)$]) / u_i∈U}, where interval [$t_A(u_i)$, 1−$f_A(u_i)$] is called the vague value of u_i in A denoted by $V_A(u_i)$

Definition 2.2:[5]

Let U be the initial universe and E be the set of parameters. Let P(U) denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U where F is a mapping given by F: A ⟶P(U)

Definition 2.3:[2]

Let U be a universe, E be a set of parameters, V(U) be the power set of vague set on U and A⊆B. A pair (F,A) is called a vague soft set over U where F is a mapping given by F:A⟶V(U). In other words, a vague soft set over U is a parameterized family of vague sets of the universe U. For e∈A, $\mu_{F(e)}$:U⟶[0,1] is regarded as the set of e-approximate elements of the vague soft sets.

Definition 2.4:[1]

Let U_1, U_2be two initial universal sets and E be a set of parameters. Let P(U_1), P(U_2) denote the power set of U_1, U_2 respectively. Also let A, B ⊆ E. A pair (F, A) is called a binary soft set over U_1,$U_2$where F is defined as below F: A⟶P(U_1)×P(U_2), F(e) = (X,Y) for each e∈A such that X ⊆U_1, Y⊆U_2

3. Vague binary soft sets

In this section vague binary sets and vague binary soft sets are introduced with some examples

Definition 3.1:

A vague binary set A in a common universe U_1,$U_2$is an object of the form

$$A=\left\{\left(\langle\frac{V_A(x_i)}{x_i};\forall\, x_i\in U_1\rangle,\langle\frac{V_A(y_i)}{y_i};\forall\, y_i\in U_2\rangle\right)\right\}$$

$V_A(x_i)$ = [$t_A(x_i)$, 1−$f_A(x_i)$]; $V_A(x_i)$:U_1⟶[0, 1] ; $V_A(y_i)$ = [$t_A(y_i)$, 1−$f_A(y_i)$]; $V_A(y_i)$:U_2⟶[0,1]

Example 3.2:

Let U_1= {b_1, b_2, b_3} and U_2= {l_1, l_2, l_3} be food varieties for break-fast and lunch respectively. Availability of these items in a particular hotel can be given using a vague binary set say A as

$$A=\left\{\left(\langle\frac{[0.3,0.5]}{b_1},\frac{[0.5,0.7]}{b_2},\frac{[0.7,0.9]}{b_3}\rangle,\langle\frac{[0.4,0.5]}{l_1},\frac{[0.6,0.8]}{l_2},\frac{[0.8,0.9]}{l_3}\rangle\right)\right\}$$

Definition 3.3:

Let U_1, U_2 be two initial universes which is common to a set E of parameters. Let V(U_1), V(U_2) denote power set of vague sets on U_1, U_2 respectively and A⊆E.

A pair ($\ddot{F}$, A) is said to be a vague binary soft set over U_1, $U_2$where $\ddot{F}$ is a mapping given by $\ddot{F}$: A ⟶V(U_1)×V(U_2) and

($\ddot{F}$, A) = {e∈A/(e, $\ddot{F}$(e))}where $\ddot{F}(e)=\left\{\left(\langle\frac{V_{\ddot{F}(e)}(x_i)}{x_i};\forall\, x_i\in U_1\rangle,\langle\frac{V_{\ddot{F}(e)}(y_i)}{y_i};\forall\, y_i\in U_2\rangle\right)\right\}$

Example 3.4:
In example 3.2. add a parameter set for characteristics of different hotels say, E={e_1 = luxury, e_2= famous, e_3= warm serviced, e_4= early check-in and late check-out, e_5= honest information, e_6= hospitality}. A customer's preferences are given by the parameter set A= {e_3,e_6}.
Vague binary soft set according to their interest is given as,

$$(\ddot{F}, \mathrm{A}) = \left\{ \begin{array}{l} \left(e_3, \left(\langle \frac{[0.3,0.5]}{b_1}, \frac{[0.5,0.7]}{b_2}, \frac{[0.7,0.9]}{b_3} \rangle, \langle \frac{[0.4,0.5]}{l_1}, \frac{[0.6,0.8]}{l_2}, \frac{[0.8,0.9]}{l_3} \rangle \right) \right), \\ \left(e_6, \left(\langle \frac{[0.4,0.6]}{b_1}, \frac{[0.6,0.8]}{b_2}, \frac{[0.8,0.9]}{b_3} \rangle, \langle \frac{[0.4,0.8]}{l_1}, \frac{[0.3,0.5]}{l_2}, \frac{[0.7,0.9]}{l_3} \rangle \right) \right) \end{array} \right\}$$

Definition 3.5:
A VBSS ($\ddot{F}$, A) over U_1, $U_2$is said to be a null vague binary soft set denoted by ($\ddot{\emptyset}$, A) if
$[t_{\ddot{F}(\mathrm{e})}(x_i), 1-f_{\ddot{F}(\mathrm{e})}(x_i)] = [0, 0]$; $\forall x_i \in U_1$, $\forall$e$\in$A;
$[t_{\ddot{F}(\mathrm{e})}(y_i), 1-f_{\ddot{F}(\mathrm{e})}(y_i)] = [0, 0]$; $\forall y_i \in U_2$, $\forall$e$\in$A

Definition 3.6:
A VBSS ($\ddot{F}$, A) over U_1, $U_2$is said to be an absolute vague binary soft set denoted by ($\ddot{U}$, A) if
$[t_{\ddot{F}(\mathrm{e})}(x_i), 1-f_{\ddot{F}(\mathrm{e})}(x_i)] = [1, 1]$; $\forall x_i \in U_1$, $\forall$e$\in$A;
$[t_{\ddot{F}(\mathrm{e})}(y_i), 1-f_{\ddot{F}(\mathrm{e})}(y_i)] = [1, 1]$; $\forall y_i \in U_2$, $\forall$e$\in$A

4. Vague binary soft set operations
Operations on vague sets are natural generalizations of corresponding operations on fuzzy sets

Definition 4.1:
Let U_1, U_2be two non-empty universal sets common to the fixed parameter set E and

$(\ddot{F}, \mathrm{A}) = \{\mathrm{e}\in\mathrm{A}/(\mathrm{e}, \ddot{F}(\mathrm{e}))\}$; $\ddot{F}(\mathrm{e}) = \left\{ \left(\langle \frac{V_{\ddot{F}(\mathrm{e})}(x_i)}{x_i}; \forall x_i \in U_1 \rangle, \langle \frac{V_{\ddot{F}(\mathrm{e})}(y_i)}{y_i}; \forall y_i \in U_2 \rangle \right) \right\}$

$(\ddot{G}, \mathrm{B}) = \{\mathrm{e}\in\mathrm{B}/(\mathrm{e}, \ddot{G}(\mathrm{e}))\}$; $\ddot{G}(\mathrm{e}) = \left\{ \left(\langle \frac{V_{\ddot{G}(\mathrm{e})}(x_i)}{x_i}; \forall x_i \in U_1 \rangle, \langle \frac{V_{\ddot{G}(\mathrm{e})}(y_i)}{y_i}; \forall y_i \in U_2 \rangle \right) \right\}$ are 2 VBSS's.

Vague binary soft union of these two sets is denoted by ($\ddot{F}$, A)$\ddot{\cup}$($\ddot{G}$, B). Let it be ($\ddot{H}$, C).
Here C = (A∪B); ∪ represents the usual set theoretic operation. Then ∀e∈C,

$$\ddot{H}(\mathrm{e}) = \begin{cases} \ddot{F}(\mathrm{e}) & ; \mathrm{e}\in(\mathrm{A}-\mathrm{B}) \\ \ddot{G}(\mathrm{e}) & ; \mathrm{e}\in(\mathrm{B}-\mathrm{A}) \\ \ddot{F}(\mathrm{e})\cup\ddot{G}(\mathrm{e}) & ; \mathrm{e}\in(\mathrm{A}\cap\mathrm{B}) \end{cases}$$

$$t_{\ddot{H}(\mathrm{e})} = \begin{cases} t_{\ddot{F}(\mathrm{e})}(x_i) & ; \mathrm{e}\in(\mathrm{A}-\mathrm{B}), \forall x_i \in U_1 \\ t_{\ddot{G}(\mathrm{e})}(x_i) & ; \mathrm{e}\in(\mathrm{B}-\mathrm{A}), \forall x_i \in U_1 \\ t_{\ddot{F}(\mathrm{e})}(y_i) & ; \mathrm{e}\in(\mathrm{A}-\mathrm{B}), \forall y_i \in U_2 \end{cases}$$

$t_{\ddot{G}(e)}(y_i)$; e∈(B−A), $\forall y_i \in U_2$

max $(t_{\ddot{F}(e)}(x_i), t_{\ddot{G}(e)}(x_i))$; e∈(A⋂B), $\forall x_i \in U_1$

max $(t_{\ddot{F}(e)}(y_i), t_{\ddot{G}(e)}(y_i))$; e∈(A⋂B), $\forall y_i \in U_2$

$1-f_{\ddot{H}(e)} = 1-f_{\ddot{F}(e)}(x_i)$; e∈(A−B), $\forall x_i \in U_1$

$1-f_{\ddot{G}(e)}(x_i)$; e∈(B−A),$\forall x_i \in U_1$

$1-f_{\ddot{F}(e)}(y_i)$; e∈(A−B), $\forall y_i \in U_2$

$1-f_{\ddot{G}(e)}(y_i)$; e∈(B−A), $\forall y_i \in U_2$

max $(1-f_{\ddot{F}(e)}(x_i), 1-f_{\ddot{G}(e)}(x_i))$; e∈(A⋂B), $\forall x_i \in U_1$

max $(1-f_{\ddot{F}(e)}(y_i), 1-f_{\ddot{G}(e)}(y_i))$; e∈(A⋂B),$\forall y_i \in U_2$

Example 4.2:
Let U_1= {b_1, b_2, b_3, b_4, b_5, b_6} be the set of books; U_2= {p_1, p_2, p_3, p_4} be the set of pencils; E= {e_1= cheap, e_2= expensive, e_3= attractive, e_4=small, e_5=long, e_6= colorful} be the set of parameters. A= {e_3, e_4, e_6} ⊆ E and B={e_2, e_3, e_5}⊆ B. Let ($\ddot{F}$, A) and ($\ddot{G}$, B) be two VBSS's over common universe U_1, U_2

Combined tabular representation of VBSS's ($\ddot{F}$, A) and ($\ddot{G}$, B) is given below

U		A			**B**		
		e_3=attractive	e_4=small	e_6=colorful	e_2=expensive	e_3=attractive	e_5=long
U_1	b_1	⟨[0.1,0.3]⟩	⟨[0.3,0.4]⟩	⟨[0.5,0.6]⟩	⟨[0.4,0.6]⟩	⟨[0.5,0.6]⟩	⟨[0.2,0.9]⟩
	b_2	⟨[0.5,0.8]⟩	⟨[0.2,0.3]⟩	⟨[0.2,0.4]⟩	⟨[0.2,0.5]⟩	⟨[0.4,0.5]⟩	⟨[0.2,0.4]⟩
	b_3	⟨[0.6,0.7]⟩	⟨[0.2,0.7]⟩	⟨[0.2,0.7]⟩	⟨[0.4,0.7]⟩	⟨[0.6,0.8]⟩	⟨[0.2,0.5]⟩
	b_4	⟨[0.2,0.6]⟩	⟨[0.4,0.8]⟩	⟨[0.4,0.8]⟩	⟨[0.2,0.9]⟩	⟨[0.3,0.6]⟩	⟨[0.2,0.6]⟩
	b_5	⟨[0.3,0.5]⟩	⟨[0.5,0.8]⟩	⟨[0.5,0.8]⟩	⟨[0.2,0.3]⟩	⟨[0.3,0.5]⟩	⟨[0.5,0.7]⟩
U_2	p_1	⟨[0.4,0.8]⟩	⟨[0.2,0.7]⟩	⟨[0.2,0.7]⟩	⟨[0.2,0.4]⟩	⟨[0.2,0.9]⟩	⟨[0.6,0.7]⟩
	p_2	⟨[0.2,0.9]⟩	⟨[0.3,0.5]⟩	⟨[0.3,0.5]⟩	⟨[0.1,0.3]⟩	⟨[0.7,0.9]⟩	⟨[0.7,0.8]⟩
	p_3	⟨[0.2,0.7]⟩	⟨[0.4,0.5]⟩	⟨[0.4,0.5]⟩	⟨[0.2,0.6]⟩	⟨[0.5,0.7]⟩	⟨[0.1,0.9]⟩

Union of VBSS's ($\ddot{F}$, A) and ($\ddot{G}$, B) is ($\ddot{H}$, C) can be represented as follows:

U \ C		e_2=expensive	e_3=attractive	e_4=small	e_5=long	e_6=colorful
U_1	b_1	⟨[0.4,0.6]⟩	⟨[0.5,0.6]⟩	⟨[0.3,0.4]⟩	⟨[0.2,0.9]⟩	⟨[0.5,0.6]⟩
	b_2	⟨[0.2,0.5]⟩	⟨[0.5,0.8]⟩	⟨[0.2,0.3]⟩	⟨[0.2,0.4]⟩	⟨[0.2,0.4]⟩
	b_3	⟨[0.4,0.7]⟩	⟨[0.6,0.8]⟩	⟨[0.2,0.7]⟩	⟨[0.2,0.5]⟩	⟨[0.2,0.7]⟩
	b_4	⟨[0.2,0.9]⟩	⟨[0.3,0.6]⟩	⟨[0.4,0.8]⟩	⟨[0.2,0.6]⟩	⟨[0.4,0.8]⟩
	b_5	⟨[0.2,0.3]⟩	⟨[0.3,0.5]⟩	⟨[0.5,0.8]⟩	⟨[0.5,0.7]⟩	⟨[0.5,0.8]⟩
U_2	p_1	⟨[0.2,0.4]⟩	⟨[0.4,0.9]⟩	⟨[0.2,0.7]⟩	⟨[0.6,0.7]⟩	⟨[0.2,0.7]⟩
	p_2	⟨[0.1,0.3]⟩	⟨[0.7,0.9]⟩	⟨[0.3,0.5]⟩	⟨[0.7,0.8]⟩	⟨[0.3,0.5]⟩
	p_3	⟨[0.2,0.6]⟩	⟨[0.5,0.7]⟩	⟨[0.4,0.5]⟩	⟨[0.1,0.9]⟩	⟨[0.4,0.5]⟩

Definition 4.3:

Let U_1,U_2 be two non-empty universal sets common to the fixed parameter set E and

($\ddot{F}$, A) = {e∈A ⁄(e, $\ddot{F}$(e))}; $\ddot{F}$(e) = $\left\{\left(\langle\frac{V_{\ddot{F}(e)}(x_i)}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{V_{\ddot{F}(e)}(y_i)}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$

($\ddot{G}$, B) = {e∈B ⁄(e, $\ddot{G}$(e))}; $\ddot{G}$(e) = $\left\{\left(\langle\frac{V_{\ddot{G}(e)}(x_i)}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{V_{\ddot{G}(e)}(y_i)}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$ are 2 VBSS's. Vague binary soft intersection of these sets is denoted by ($\ddot{F}$, A)$\ddot{\cap}$($\ddot{G}$, B). Let it be ($\ddot{K}$, D). Here D = (A∩B); ∩ represents the usual set theoretic operation. Then ∀e∈ D,

$\ddot{K}$(e) = $\ddot{F}$(e) ; e∈ (A−B)

$\ddot{G}$(e) ; e∈ (B−A)

$\ddot{F}$(e)∩$\ddot{G}$(e) ; e∈ (A∩B)

$t_{\ddot{K}(e)}$ = $t_{\ddot{F}(e)}(x_i)$; e∈ (A−B), $\forall x_i \in U_1$

$t_{\ddot{G}(e)}(x_i)$; e∈ (B−A), $\forall x_i \in U_1$

$t_{\ddot{F}(e)}(y_i)$; e∈ (A−B), $\forall y_i \in U_2$

$t_{\ddot{G}(e)}(y_i)$; e∈ (B−A), $\forall y_i \in U_2$

$\min(t_{\ddot{F}(e)}(x_i), t_{\ddot{G}(e)}(x_i))$; e∈ (A∩B), $\forall x_i \in U_1$

$\min(t_{\ddot{F}(e)}(y_i), t_{\ddot{G}(e)}(y_i))$; e∈ (A∩B), $\forall y_i \in U_2$

$1-f_{\ddot{K}(e)}$ = $1-f_{\ddot{F}(e)}(x_i)$; e∈ (A−B), $\forall x_i \in U_1$

$1-f_{\ddot{G}(e)}(x_i)$; $e\in(B-A), \forall x_i \in U_1$

$1-f_{\ddot{F}(e)}(y_i)$; $e\in(A-B), \forall y_i \in U_2$

$1-f_{\ddot{G}(e)}(y_i)$; $e\in(B-A), \forall y_i \in U_2$

$\min(1-f_{\ddot{F}(e)}(x_i), 1-f_{\ddot{G}(e)}(x_i))$; $e\in(A\cap B), \forall x_i \in U_1$

$\min(1-f_{\ddot{F}(e)}(y_i), 1-f_{\ddot{G}(e)}(y_i))$; $e\in(A\cap B), \forall y_i \in U_2$

Example 4.4:

Consider the example given in 4.2.

Intersection of VBSS's ($\ddot{F}$, A) and ($\ddot{G}$, B) is ($\ddot{K}$, D) can be represented as follows:

U \ D		e_2=expensive	e_3=attractive	e_4=small	e_5=long	e_6=colorful
U_1	b_1	⟨[0.4,0.6]⟩	⟨[0.1,0.3]⟩	⟨[0.3,0.4]⟩	⟨[0.2,0.9]⟩	⟨[0.5,0.6]⟩
	b_2	⟨[0.2,0.5]⟩	⟨[0.4,0.5]⟩	⟨[0.2,0.3]⟩	⟨[0.2,0.4]⟩	⟨[0.2,0.4]⟩
	b_3	⟨[0.4,0.7]⟩	⟨[0.6,0.7]⟩	⟨[0.2,0.7]⟩	⟨[0.2,0.5]⟩	⟨[0.2,0.7]⟩
	b_4	⟨[0.2,0.9]⟩	⟨[0.2,0.6]⟩	⟨[0.4,0.8]⟩	⟨[0.2,0.6]⟩	⟨[0.4,0.8]⟩
	b_5	⟨[0.2,0.3]⟩	⟨[0.3,0.5]⟩	⟨[0.5,0.8]⟩	⟨[0.5,0.7]⟩	⟨[0.5,0.8]⟩
U_2	p_1	⟨[0.2,0.4]⟩	⟨[0.2,0.8]⟩	⟨[0.2,0.7]⟩	⟨[0.6,0.7]⟩	⟨[0.2,0.7]⟩
	p_2	⟨[0.1,0.3]⟩	⟨[0.2,0.9]⟩	⟨[0.3,0.5]⟩	⟨[0.7,0.8]⟩	⟨[0.3,0.5]⟩
	p_3	⟨[0.2,0.6]⟩	⟨[0.2,0.7]⟩	⟨[0.4,0.5]⟩	⟨[0.1,0.9]⟩	⟨[0.4,0.5]⟩

Definition 4.5:

Let ($\ddot{F}$, A) and ($\ddot{G}$, B) be two vague binary soft sets over a common universe U_1, U_2 with a fixed set E of parameters. AND operation between these sets is denoted as ($\ddot{F}$, A)$\ddot{\wedge}$($\ddot{G}$, B). Let it be ($\ddot{M}$, A×B) ; $\ddot{M}$: A×B→V(U_1)×V(U_2) and $\ddot{M}$(a, b) = $\ddot{F}$(a)∩$\ddot{G}$(b), ∀(a, b)∈C=(A×B).

Here ∩ denotes the usual set theoretical operation

Example 4.6:

Let U_1= {b_1, b_2, b_3, b_4, b_5, b_6} be the set of buses; U_2= {r_1, r_2, r_3} be the set of routes; E= {e_1= rural, e_2= urban, e_3= inter-state, e_4= air-bus, e_5= luxury, e_6= sleeper} be the set of parameters. A= {e_2= urban, e_4= air-bus}; B = {e_3= inter-state}; C= A×B = {(e_2, e_3), (e_4, e_3)}.

Combined tabular representation of VBSS's ($\ddot{F}$, A) and ($\ddot{G}$, B) is given as follows:

U		A		B
		e_2= urban	e_4= air-bus	e_3=inter-state
U_1	b_1	⟨[0.1,0.3]⟩	⟨[0.3,0.4]⟩	⟨[0.5,0.6]⟩
	b_2	⟨[0.5,0.8]⟩	⟨[0.2,0.3]⟩	⟨[0.4,0.5]⟩
	b_3	⟨[0.6,0.7]⟩	⟨[0.2,0.7]⟩	⟨[0.6,0.8]⟩
	b_4	⟨[0.2,0.6]⟩	⟨[0.4,0.8]⟩	⟨[0.3,0.6]⟩
	b_5	⟨[0.3,0.5]⟩	⟨[0.5,0.8]⟩	⟨[0.3,0.5]⟩
U_2	r_1	⟨[0.4,0.8]⟩	⟨[0.2,0.7]⟩	⟨[0.2,0.9]⟩
	r_2	⟨[0.2,0.9]⟩	⟨[0.3,0.5]⟩	⟨[0.7,0.9]⟩
	r_3	⟨[0.2,0.7]⟩	⟨[0.4,0.5]⟩	⟨[0.5,0.7]⟩

AND operation of ($\ddot{F}$, A) and ($\ddot{G}$, B) is VBSS ($\ddot{M}$, A×B) is as follows:

C =A×B / U		(e_2, e_3)=(urban, inter-state)	(e_4, e_3)=(air-bus, inter-state)
U_1	b_1	⟨[0.1,0.3]⟩	⟨[0.3,0.4]⟩
	b_2	⟨[0.4,0.5]⟩	⟨[0.2,0.3]⟩
	b_3	⟨[0.6,0.7]⟩	⟨[0.2,0.7]⟩
	b_4	⟨[0.2,0.6]⟩	⟨[0.3,0.6]⟩
	b_5	⟨[0.3,0.5]⟩	⟨[0.3,0.5]⟩
U_2	r_1	⟨[0.2,0.8]⟩	⟨[0.2,0.7]⟩
	r_2	⟨[0.2,0.9]⟩	⟨[0.3,0.5]⟩
	r_3	⟨[0.2,0.7]⟩	⟨[0.4,0.5]⟩

Definition 4.7:
Let ($\ddot{F}$, A) and ($\ddot{G}$, B) be two VBSS's over a common universe U_1, U_2 with a set E of parameters. OR operation between these sets is denoted as ($\ddot{F}$, A)$\ddot{\vee}$($\ddot{G}$, B). Let it be ($\ddot{N}$, A×B) ; $\ddot{N}$: A×B →V(U_1)×V(U_2) and $\ddot{M}$(a, b) = $\ddot{F}$(a)∪$\ddot{G}$(b), ∀(a, b)∈ C = (A×B).
Here ∪ denotes the usual set theoretical operation

Example 4.8:

Consider example 4.6: OR operation of ($\ddot{F}$, A) and ($\ddot{G}$, B) is VBSS ($\ddot{N}$, A×B) is as follows:

U \ C =A×B		(e_2, e_3) =(urban, inter-state)	(e_4, e_3) =(air-bus, inter-state)
U_1	b_1	⟨[0.5,0.6]⟩	⟨[0.5,0.6]⟩
	b_2	⟨[0.5,0.8]⟩	⟨[0.4,0.5]⟩
	b_3	⟨[0.6,0.8]⟩	⟨[0.6,0.8]⟩
	b_4	⟨[0.3,0.6]⟩	⟨[0.4,0.8]⟩
	b_5	⟨[0.3,0.5]⟩	⟨[0.5,0.8]⟩
U_2	r_1	⟨[0.4,0.9]⟩	⟨[0.2,0.9]⟩
	r_2	⟨[0.7,0.9]⟩	⟨[0.7,0.9]⟩
	r_3	⟨[0.5,0.7]⟩	⟨[0.5,0.7]⟩

Definition 4.9:
A vague binary soft set ($\ddot{F}$, A) is contained in another vague binary soft set ($\ddot{G}$, B) denoted as ($\ddot{F}$, A)$\ddot{\sqsubseteq}$ ($\ddot{G}$, B) if
(1) A⊆B
(2) ∀ e∈ A, $\ddot{F}$(e) and $\ddot{G}$(e) are identical approximations
In this case ($\ddot{F}$, A) is called *vague binary soft subset* of ($\ddot{G}$, B) and ($\ddot{G}$, B) is called vague binary soft superset of ($\ddot{F}$, A) where ⊆ denotes the usual set theoretical operation
Example 4.10:
Let $U_1 = \{b_1, b_2, b_3\}$; $U_2 = \{p_1, p_2, p_3\}$; E= $\{e_1, e_2, e_3, e_4\}$;
A= $\{e_3, e_4\}$ ⊆ E & B= $\{e_2, e_3, e_4\}$⊆E.
Let ($\ddot{F}$, A) & ($\ddot{G}$, B) be two VBSS's over common universe U_1, U_2.

$$(\ddot{F}, A) = \{(e_3, \langle\frac{[0.2,0.3]}{b_1}, \frac{[0.4,0.6]}{b_2}, \frac{[0.2,0.8]}{b_3}\rangle, \langle\frac{[0.5,0.6]}{p_1}, \frac{[0.2,0.6]}{p_2}, \frac{[0.8,0.9]}{p_3}\rangle),$$

$$(e_4, \langle\frac{[0.3,0.4]}{b_1}, \frac{[0.5,0.7]}{b_2}, \frac{[0.5,0.6]}{b_3}\rangle, \langle\frac{[0.7,0.8]}{p_1}, \frac{[0.4,0.7]}{p_2}, \frac{[0.2,0.8]}{p_3}\rangle)\}$$

$$(\ddot{G}, B) = \{(e_2, \langle\frac{[0.4,0.5]}{b_1}, \frac{[0.5,0.6]}{b_2}, \frac{[0.3,0.5]}{b_3}\rangle, \langle\frac{[0.6,0.6]}{p_1}, \frac{[0.3,0.6]}{p_2}, \frac{[0.7,0.8]}{p_3}\rangle),$$

$$(e_3, \langle\frac{[0.2,0.3]}{b_1}, \frac{[0.4,0.6]}{b_2}, \frac{[0.2,0.8]}{b_3}\rangle, \langle\frac{[0.5,0.6]}{p_1}, \frac{[0.2,0.6]}{p_2}, \frac{[0.8,0.9]}{p_3}\rangle)$$

$$(e_4, \langle\frac{[0.3,0.4]}{b_1}, \frac{[0.5,0.7]}{b_2}, \frac{[0.5,0.6]}{b_3}\rangle, \langle\frac{[0.7,0.8]}{p_1}, \frac{[0.4,0.7]}{p_2}, \frac{[0.2,0.8]}{p_3}\rangle)\}$$

In this case ($\ddot{F}$, A)$\ddot{\sqsubseteq}$ ($\ddot{G}$, B) i.e., ($\ddot{F}$, A) is a vague binary soft subset of ($\ddot{G}$, B).

$(\ddot{G}, B) \ddot{\supseteq} (\ddot{F}, A)$ i.e., $(\ddot{G}, B)$ is a vague binary soft superset of $(\ddot{F}, A)$

Definition 4.11:

Let $(\ddot{F}, A)$ and $(\ddot{G}, B)$ are two VBSS's. These sets are said to be *vague binary soft equal* if $(\ddot{F}, A)$ is a *vague binary soft subset* of $(\ddot{G}, B)$ and $(\ddot{G}, B)$ is a vague binary soft superset of $(\ddot{F}, A)$. It is denoted by $(\ddot{F}, A) \ddot{=} (\ddot{G}, B)$

Definition 4.12:

Complement of a VBSS $(\ddot{F}, A)$ with respect to absolute VBSS $(\ddot{U}, A)$ is denoted by $(\ddot{F}, A)^c = (\ddot{F}^c, A)$ is a mapping given by $\ddot{F}^c$: $A \longrightarrow V(U_1) \times V(U_2)$ such that

$$t_{\ddot{F}^c(e)}(x_i) = f_{\ddot{F}(e)}(x_i) \quad ; \forall e \in A, \forall x_i \in U_1$$

$$t_{\ddot{F}^c(e)}(y_i) = f_{\ddot{F}(e)}(y_i) \quad ; \forall e \in A, \forall y_i \in U_2$$

$$1 - f_{\ddot{F}^c(e)}(x_i) = 1 - t_{\ddot{F}(e)}(x_i) \; ; \forall e \in A, \forall x_i \in U_1$$

$$1 - f_{\ddot{F}^c(e)}(y_i) = 1 - t_{\ddot{F}(e)}(y_i) \; ; \forall e \in A, \forall y_i \in U_2$$

Example 4.13:

Let $U_1 = \{b_1, b_2, b_3, b_4, b_5, b_6\}$ be the set of books; $U_2 = \{p_1, p_2, p_3, p_4\}$ be the set of pencils; $E = \{e_1 =$ cheap, $e_2 =$ expensive, $e_3 =$ attractive, $e_4 =$ small, $e_5 =$ long, $e_6 =$ colorful$\}$ be the set of parameters. The binary soft set $(\ddot{F}, A)$ where $A = \{e_2, e_6\}$describes expensive & colorful features of both books and pencils which a stationary shop proprietor is going to purchase from a wholesale dealer. $(\ddot{F}, A)$ is a VBSS over U_1, U_2 defined as follows.

$$\ddot{F}(e_2) = \left\{ \left\langle \frac{[0.2,0.3]}{b_1}, \frac{[0.4,0.6]}{b_2}, \frac{[0.2,0.8]}{b_3}, \frac{[0.5,0.8]}{b_4} \frac{[0.7,0.8]}{b_5} \right\rangle, \left\langle \frac{[0.5,0.6]}{p_1}, \frac{[0.2,0.6]}{p_2}, \frac{[0.8,0.9]}{p_3}, \frac{[0.4,0.7]}{p_4} \right\rangle \right\}$$

$$\ddot{F}(e_6) = \left\{ \left\langle \frac{[0.3,0.4]}{b_1}, \frac{[0.5,0.7]}{b_2}, \frac{[0.5,0.6]}{b_3}, \frac{[0.6,0.7]}{b_4} \frac{[0.8,0.9]}{b_5} \right\rangle, \left\langle \frac{[0.7,0.8]}{p_1}, \frac{[0.4,0.7]}{p_2}, \frac{[0.2,0.8]}{p_3}, \frac{[0.5,0.5]}{p_4} \right\rangle \right\}$$

The complement of the above vague binary soft set is given by $(\ddot{F}, A)^c$ is given by

$$\ddot{F}^c(e_2) = \left\{ \left\langle \frac{[0.7,0.8]}{b_1}, \frac{[0.4,0.6]}{b_2}, \frac{[0.2,0.8]}{b_3}, \frac{[0.2,0.5]}{b_4} \frac{[0.2,0.3]}{b_5} \right\rangle, \left\langle \frac{[0.4,0.5]}{p_1}, \frac{[0.4,0.8]}{p_2}, \frac{[0.1,0.2]}{p_3}, \frac{[0.3,0.6]}{p_4} \right\rangle \right\}$$

$$\ddot{F}^c(e_6) = \left\{ \left\langle \frac{[0.6,0.7]}{b_1}, \frac{[0.3,0.5]}{b_2}, \frac{[0.4,0.5]}{b_3}, \frac{[0.3,0.4]}{b_4} \frac{[0.1,0.2]}{b_5} \right\rangle, \left\langle \frac{[0.2,0.3]}{p_1}, \frac{[0.3,0.6]}{p_2}, \frac{[0.2,0.8]}{p_3}, \frac{[0.5,0.5]}{p_4} \right\rangle \right\}$$

Definition 4.14:

Let $(\ddot{F}, A)$ and $(\ddot{G}, B)$ be two VBSS's over U_1, U_2.
Cartesian product of $(\ddot{F}, A)$ and $(\ddot{G}, B)$ is denoted as $(\ddot{F}, A) \ddot{\times} (\ddot{G}, B)$. Let it be $(\ddot{H}, A \times B)$.
It is a mapping given by $\ddot{H}$: $A \times B \longrightarrow V(U_1 \times U_1) \times V(U_2 \times U_2)$;
$\ddot{H}(a, b) = \ddot{F}(a) \times \ddot{G}(b); \forall (a, b) \in (A \times B)$.

$$\ddot{H}(a,b)=\left\{ \begin{array}{l} \langle\frac{[\min(t_{\ddot{F}(e)}(x_i),t_{\ddot{G}(e)}(x_i)),\ \max(1-f_{\ddot{F}(e)}(x_i),1-f_{\ddot{G}(e)}(x_i))]}{x_i}; \forall x_i \in U_1\rangle, \\ \langle\frac{[\min(t_{\ddot{F}(e)}(y_i),t_{\ddot{G}(e)}(y_i)),\ \max(1-f_{\ddot{F}(e)}(y_i),1-f_{\ddot{G}(e)}(y_i))]}{y_i}; \forall y_i \in U_2\rangle \end{array}\right\}$$

Example 4.15:

$U_1=\{t_1, t_2\}$ be the set T-shirts; $U_2=\{k_1, k_2\}$ be the set of kurthas.
E= $\{e_1, e_2, e_3, e_4, e_5, e_6\}$ be the set of parameters.
A=$\{e_1, e_2\}\subseteq$E; B=$\{e_3, e_4\}\subseteq$E; Let ($\ddot{F}$, A) and ($\ddot{G}$, B) be two vague binary soft sets over U_1, U_2.

$$(\ddot{F}, A) = \{(e_1, (\langle\frac{[0.2,0.3]}{t_1},\frac{[0.4,0.6]}{t_2}\rangle,\langle\frac{[0.5,0.6]}{k_1},\frac{[0.2,0.6]}{k_2}\rangle)), (e_2, (\langle\frac{[0.3,0.4]}{t_1},\frac{[0.5,0.7]}{t_2}\rangle,\langle\frac{[0.7,0.8]}{k_1},\frac{[0.4,0.7]}{k_2}\rangle))\}$$

$$(\ddot{G}, B) = \{(e_3, (\langle\frac{[0.4,0.5]}{t_1},\frac{[0.8,0.9]}{t_2}\rangle,\langle\frac{[0.4,0.6]}{k_1},\frac{[0.3,0.6]}{k_2}\rangle)), (e_4, (\langle\frac{[0.5,0.7]}{t_1},\frac{[0.4,0.8]}{t_2}\rangle,\langle\frac{[0.6,0.8]}{k_1},\frac{[0.3,0.7]}{k_2}\rangle))\}$$

C=A×B=$\{(e_1,e_3), (e_1, e_4), (e_2, e_3), (e_2,e_4)\}$

$(\ddot{F}, A)\ \ddot{\times}(\ddot{G}, B) = (\ddot{H}, C)= (\ddot{H}, A\times B)=\{\ddot{H}(e_1,e_3),\ddot{H}(e_1, e_4), \ddot{H}(e_2, e_3),\ddot{H}(e_2, e_4)\}$

$$\ddot{H}(e_1, e_3)=\{\left((e_1, e_3), \langle\frac{[0.2,0.5]}{(t_1,t_1)},\frac{[0.2,0.9]}{(t_1,t_2)}\rangle,\langle\frac{[0.2,0.6]}{(t_1,k_1)},\frac{[0.2,0.6]}{(t_1,k_2)}\rangle\right),$$

$$\left((e_1, e_3), (\langle\frac{[0.4,0.6]}{(t_2,t_1)},\frac{[0.4,0.9]}{(t_2,t_2)}\rangle,\langle\frac{[0.4,0.6]}{(t_2,k_1)},\frac{[0.3,0.6]}{(t_2,k_2)}\rangle)\right), \left((e_1, e_3), (\langle\frac{[0.2,0.5]}{(t_1,t_1)},\frac{[0.2,0.9]}{(t_1,t_2)}\rangle,\langle\frac{[0.5,0.6]}{(t_1,k_1)},\frac{[0.2,0.6]}{(t_1,k_2)}\rangle)\right),$$

$$\left((e_1, e_3), (\langle\frac{[0.2,0.5]}{(k_1,t_1)},\frac{[0.2,0.9]}{(k_1,t_2)}\rangle,\langle\frac{[0.5,0.6]}{(k_1,k_1)},\frac{[0.2,0.6]}{(k_1,k_2)}\rangle)\right), \left((e_1, e_3), (\langle\frac{[0.2,0.5]}{(k_2,t_1)},\frac{[0.2,0.9]}{(k_2,t_2)}\rangle,\langle\frac{[0.5,0.6]}{(k_2,k_1)},\frac{[0.2,0.6]}{(k_2,k_2)}\rangle)\right)\}$$

5. Algebraic properties of vague binary soft set operations

Some of the algebraic properties for the vague binary soft set operations defined in the last section are discussed here.

Proposition 5.1: (Identity laws)

For any vague binary soft set ($\ddot{F}$, A) defined on the absolute vague binary soft set ($\ddot{U}$, E)
(1) ($\ddot{F}$, A)∪($\ddot{\emptyset}$, A) = ($\ddot{F}$, A)
(2)($\ddot{F}$, A)∩($\ddot{U}$, A) = ($\ddot{F}$, A)

Proof

(1) Let($\ddot{F}$, A) ={e∈A/(e, $\ddot{F}$(e))} where $\ddot{F}(e) = \{((\langle\frac{V_{\ddot{F}(e)}(x_i)}{x_i}\forall x_i \in U_1\rangle,\langle\frac{V_{\ddot{F}(e)}(y_i)}{y_i}; \forall x_i \in$
□2

$$(\ddot{\emptyset}, A) = \{e\in A/(e, \ddot{\emptyset}(e))\} \text{ where } \ddot{\emptyset}(e) = \{((\langle\frac{[0,0]}{x_i}; \forall x_i \in U_1\rangle,\langle\frac{[0,0]}{y_i}; \forall y_i \in U_2\rangle))\}$$

$$(\ddot{F}, A)\cup(\ddot{\emptyset}, A) = \{e\in A /(e,\{((\langle\frac{\max(V_{\ddot{F}(e)}(x_i),[0,0])}{x_i}\rangle; \forall x_i \in U_1\rangle,\langle\frac{\max(V_{\ddot{F}(e)}(y_i),[0,0])}{y_i}\rangle; \forall y_i \in U_2\rangle))\}$$

$$= \{e \in A \,/(e, \left\{\left(\langle\frac{V_{\ddot{F}(e)}(x_i)}{x_i}\rangle; \forall x_i \in U_1\rangle, \langle\frac{V_{\ddot{F}(e)}(yi)}{yi}\rangle; \forall y_i \in U_2\rangle\right)\right\} = (\ddot{F}, A)$$

(2) Proof is similar to (1)

Proposition 5.2:(Domination laws)

For any vague binary soft set ($\ddot{F}$, A) defined on the absolute vague binary soft set ($\ddot{U}$, A)

(1) ($\ddot{F}$, A)∩($\ddot{\emptyset}$, A)=($\ddot{\emptyset}$, A)

(2) ($\ddot{F}$, A)∪($\ddot{U}$, A)= ($\ddot{U}$, A)

Proof

(1) Let($\ddot{F}$, A) = {e∈ A /(e, $\ddot{F}$(e))} where $\ddot{F}(e)=\left\{\left(\langle\frac{V_{\ddot{F}(e)}(x_i)}{xi}; \forall x_i \in U_1\rangle, \langle\frac{V_{\ddot{F}(e)}(y_i)}{y_i}; \forall y_i \in\right.\right.$
□2

$$(\ddot{\emptyset}, A) = \{e \in A \,/(e, \ddot{\emptyset}(e))\} = \left\{\left(\langle\frac{[0,0]}{x_i}\rangle; \forall x_i \in U_1\rangle, \langle\frac{[0,0]}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$$

$$(\ddot{F}, A)\cap(\ddot{\emptyset}, A) = \left\{\left(\langle\frac{\min(V_{\ddot{F}(e)}(x_i),[0,0])}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{\min(V_{\ddot{F}(e)}(y_i),[0,0])}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$$

$$= \left\{\left(\langle\frac{[0,0]}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{[0,0]}{y_i}; \forall y_i \in U_2\rangle\right)\right\} = (\ddot{\emptyset}, A)$$

(2) Proof is similar to (1)

Proposition 5.3:(Idempotent laws)

For any vague binary soft set ($\ddot{F}$, A) over the absolute vague binary soft set ($\ddot{U}$, A)

(1) ($\ddot{F}$, A)∩($\ddot{F}$, A)= ($\ddot{F}$, A)

(2)($\ddot{F}$, A)∪($\ddot{F}$, A) = ($\ddot{F}$, A)

Proof

(1) Let ($\ddot{F}$, A) = {e∈ A/(e, $\ddot{F}$(e))} where $\ddot{F}(e) = \left\{\left(\langle\frac{V_{\ddot{F}(e)}(x_i)}{x_i}\rangle; \forall x_i \in U_1\rangle, \langle\frac{V_{\ddot{F}(e)}(y_i)}{y_i}\rangle; \forall y_i \in\right.\right.$
□2⟩

$$(\ddot{F}, A)\cap(\ddot{F}, A) = \left\{\left(\langle\frac{\min(V_{\ddot{F}(e)}(x_i),V_{\ddot{F}(e)}(x_i))}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{\min(V_{\ddot{F}(e)}(y_i),V_{\ddot{F}(e)}(y_i))}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$$

$$= \left\{\left(\langle\frac{V_{\ddot{F}(e)}(x_i)}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{V_{\ddot{F}(e)}(y_i)}{yi}; \forall y_i \in U_2\rangle\right)\right\} = (\ddot{F}, A)$$

(2) Proof is similar to (1)

Proposition 5.4:(Complemental laws)

(1) $(\ddot{\emptyset}, A)^c = (\ddot{U}, A)$

(2) $(\ddot{U}, A)^c = (\ddot{\emptyset}, A)$

Proof

(1) Let ($\ddot{\emptyset}$, A) = ($\ddot{F}$, A)

$(\ddot{\emptyset}, A) = \{e \in A/ (e, \left\{\left(\langle\frac{[0,0]}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{[0,0]}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$

$(\ddot{F}, A) = \{e \in A \,/\, (e, \left\{\left(\langle\frac{V_{\ddot{F}(e)}(x_i)}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{V_{\ddot{F}(e)}(y_i)}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$

Then $\forall e \in A$, $\ddot{F}(e) = \left\{\left(\langle\frac{V_{\ddot{F}(e)}(x_i)}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{V_{\ddot{F}(e)}(y_i)}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$

$$= \left\{\left(\langle\frac{[0,0]}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{[0,0]}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$$

$(\ddot{F}(e))^c = \left\{\left(\langle\frac{[0,0]}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{[0,0]}{y_i}; \forall y_i \in U_2\rangle\right)\right\}^c$

$$= \left\{\left(\langle\frac{[1,1]}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{[1,1]}{y_i}; \forall y_i \in U_2\rangle\right)\right\} = \ddot{U}(e), \forall e \in A \Longrightarrow (\ddot{\emptyset}, A)^c = (\ddot{U}, A)$$

(2) Proof is similar to (1)

Proposition 5.5:(Commutative laws)

For any vague binary soft set $(\ddot{F}, A)$ and $(\ddot{G}, B)$ over the absolute vague binary soft set $(\ddot{U}, E)$

(1) $(\ddot{F}, A) \cap (\ddot{G}, B) = (\ddot{G}, B) \cap (\ddot{F}, A)$

(2) $(\ddot{F}, A) \cup (\ddot{G}, B) = (\ddot{G}, B) \cup (\ddot{F}, A)$

Proof

Proof is obvious

Proposition 5.6:(Associative laws)

For any vague binary soft set $(\ddot{F}, A)$, $(\ddot{G}, B)$ and $(\ddot{H}, C)$ over absolute vague binary soft set $(\ddot{U}, E)$

(1) $((\ddot{F}, A) \cap (\ddot{G}, B)) \cap (\ddot{H}, C) = (\ddot{F}, A) \cap ((\ddot{G}, B) \cap (\ddot{H}, C))$

(2) $((\ddot{F}, A) \cup (\ddot{G}, B)) \cup (\ddot{H}, C) = (\ddot{F}, A) \cup ((\ddot{G}, B) \cup (\ddot{H}, C))$

Proof

Proof is obvious

Proposition 5.7:(Distributive laws)

(1) $(\ddot{F}, A) \cap ((\ddot{G}, B)) \cup (\ddot{H}, C)) = ((\ddot{F}, A) \cap (\ddot{G}, B)) \cup ((\ddot{F}, A)) \cap (\ddot{H}, C))$

(2) $(\ddot{F}, A) \cup ((\ddot{G}, B))) \cap (\ddot{H}, C))) = ((\ddot{F}, A) \cup (\ddot{G}, B))) \cap ((\ddot{F}, A) \cup (\ddot{H}, C))$

Proof

Proof is obvious

Proposition 5.8:

Let $(\ddot{F}, A)$ and $(\ddot{G}, A)$ be two vague binary soft sets in $VBSS(U_1, U_2)_A$. Then following are true.

(1) $(\ddot{F}, A) \subseteq (\ddot{G}, A) \Leftrightarrow (\ddot{F}, A) \cap (\ddot{G}, A) = (\ddot{F}, A)$

(2) $(\ddot{F}, A) \subseteq (\ddot{G}, A) \Leftrightarrow (\ddot{F}, A) \cup (\ddot{G}, A) = (\ddot{F}, A)$

Proof

Let $(\ddot{F}, A) \subseteq (\ddot{G}, A)$

Then $\ddot{F}(e) \subseteq \ddot{G}(e)$, $\forall e \in A$

Let $(\ddot{H}, A) = (\ddot{F}, A) \cap (\ddot{G}, A)$

Since $\ddot{H}(e) = \ddot{F}(e) \cap \ddot{G}(e)$, $\forall e \in A \Longrightarrow \ddot{H}(e) = \ddot{F}(e)$, $\forall e \in A$

By definition $(\ddot{H}, A) = (\ddot{F}, A)$

Again suppose that $(\ddot{F}, A) \cap (\ddot{G}, A) = (\ddot{F}, A)$

Let $(\ddot{F}, A) \cap (\ddot{G}, A) = (\ddot{H}, A)$

Since $\ddot{H}(e) = \ddot{F}(e) \cap \ddot{G}(e)$, $\forall e \in A \Rightarrow \ddot{H}(e) \subseteq \ddot{G}(e)$, $\forall e \in A \Rightarrow (\ddot{F}, A) \subseteq (\ddot{G}, A)$

(2) Proof is similar to (1)

Proposition 5.9:

For any vague binary soft set $(\ddot{F}, A)$ and $(\ddot{G}, B)$ over absolute vague binary soft set $(\ddot{U}, E)$

(1) $((\ddot{F}, A) \ddot{\cup} (\ddot{G}, B))^c \subseteq (\ddot{F}, A)^c \cup (\ddot{G}, B)^c$

(2) $(\ddot{F}, A)^c \ddot{\cap} (\ddot{G}, B)^c \subseteq ((\ddot{F}, A) \cap (\ddot{G}, B))^c$

Proof

(1) Let $(\ddot{F}, A) \ddot{\cup} (\ddot{G}, B) = (\ddot{H}, C)$ where $C=(A \cup B)$ and $\forall e \in C$,

$$\ddot{H}(e) = \begin{cases} \ddot{F}(e) & ; e \in (A-B) \\ \ddot{G}(e) & ; e \in (B-A) \\ \ddot{F}(e) \ddot{\cup} \ddot{G}(e) & ; e \in (A \cap B) \end{cases}$$

$$t_{\ddot{H}(e)} = \begin{cases} t_{\ddot{F}(e)}(x_i) & ; e \in (A-B), \forall x_i \in U_1 \\ t_{\ddot{G}(e)}(x_i) & ; e \in (B-A), \forall x_i \in U_1 \\ t_{\ddot{F}(e)}(y_i) & ; e \in (A-B), \forall y_i \in U_2 \\ t_{\ddot{G}(e)}(y_i) & ; e \in (B-A), \forall y_i \in U_2 \\ \max(t_{\ddot{F}(e)}(x_i), t_{\ddot{G}(e)}(x_i)) & ; e \in (A \cap B), \forall x_i \in U_1 \\ \max(t_{\ddot{F}(e)}(y_i), t_{\ddot{G}(e)}(y_i)) & ; e \in (A \cap B), \forall y_i \in U_2 \end{cases}$$

$$1-f_{\ddot{H}(e)} = \begin{cases} 1-f_{\ddot{F}(e)}(x_i) & ; e \in (A-B), \forall x_i \in U_1 \\ 1-f_{\ddot{G}(e)}(x_i) & ; e \in (B-A), \forall x_i \in U_1 \\ 1-f_{\ddot{F}(e)}(y_i) & ; e \in (A-B), \forall y_i \in U_2 \\ 1-f_{\ddot{G}(e)}(y_i) & ; e \in (B-A), \forall y_i \in U_2 \\ \max(1-f_{\ddot{F}(e)}(x_i, 1-f_{\ddot{G}(e)}(x_i) & ; e \in (A \cap B), \forall x_i \in U_1 \\ \max(1-f_{\ddot{F}(e)}(y_i), 1-f_{\ddot{G}(e)}(y_i)); & e \in (A \cap B), \forall y_i \in U_2 \end{cases}$$

$((\ddot{F}, A) \ddot{\cup} (\ddot{G}, B))^c = (\ddot{H}, C)^c$; $C=(A \cup B)$ and $\forall e \in C$,

$\ddot{H}^c(e) = \ddot{F}^c(e)$; $e \in (A-B)$

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$\ddot{\mathcal{G}}^c(e)$; $e \in (B-A)$

$\ddot{\mathcal{F}}^c(e) \cup \ddot{\mathcal{G}}^c(e)$; $e \in (A \cap B)$

$t_{\ddot{\mathcal{H}}(e)} = f_{\ddot{\mathcal{F}}(e)}(x_i)$; $e \in (A-B), \forall x_i \in U_1$

$f_{\ddot{\mathcal{G}}(e)}(x_i)$; $e \in (B-A), \forall x_i \in U_1$

$f_{\ddot{\mathcal{F}}(e)}(y_i)$; $e \in (A-B), \forall y_i \in U_2$

$f_{\ddot{\mathcal{G}}(e)}(y_i)$; $e \in (B-A), \forall y_i \in U_2$

$\max(f_{\ddot{\mathcal{F}}(e)}(x_i), f_{\ddot{\mathcal{G}}(e)}(x_i))$; $e \in (A \cap B), \forall x_i \in U_1$

$\max(f_{\ddot{\mathcal{F}}(e)}(y_i), f_{\ddot{\mathcal{G}}(e)}(y_i))$; $e \in (A \cap B), \forall y_i \in U_2$

$1 - f_{\ddot{\mathcal{H}}(e)} = 1 - t_{\ddot{\mathcal{F}}(e)}(x_i)$; $e \in (A-B), \forall x_i \in U_1$

$1 - t_{\ddot{\mathcal{G}}(e)}(x_i)$; $e \in (B-A), \forall x_i \in U_1$

$1 - t_{\ddot{\mathcal{F}}(e)}(y_i)$; $e \in (A-B), \forall y_i \in U_2$

$1 - t_{\ddot{\mathcal{G}}(e)}(y_i)$; $e \in (B-A), \forall y_i \in U_2$

$\max((1 - f_{\ddot{\mathcal{F}}(e)}(x_i), 1 - f_{\ddot{\mathcal{G}}(e)}(x_i))$; $e \in (A \cap B), \forall x_i \in U_1$

$\max((1 - f_{\ddot{\mathcal{F}}(e)}(y_i), 1 - f_{\ddot{\mathcal{G}}(e)}(y_i))$; $e \in (A \cap B), \forall y_i \in U_2$

Let $(\ddot{\mathcal{F}}, A)^c \cup (\ddot{\mathcal{G}}, B)^c = (M, N)$ where $N = (A \cup B)$ and $\forall e \in N$,

$\ddot{\mathcal{M}}(e) = \ddot{\mathcal{F}}^c(e)$; $e \in (A-B)$

$\ddot{\mathcal{G}}^c(e)$; $e \in (B-A)$

$\ddot{\mathcal{F}}^c(e) \cup \ddot{\mathcal{G}}^c(e)$; $e \in (A \cap B)$

$t_{\ddot{\mathcal{M}}c(e)} = f_{\ddot{\mathcal{F}}(e)}(x_i)$; $e \in (A-B), \forall x_i \in U_1$

$f_{\ddot{\mathcal{G}}(e)}(x_i)$; $e \in (B-A), \forall x_i \in U_1$

$f_{\ddot{\mathcal{F}}(e)}(y_i)$; $e \in (A-B), \forall y_i \in U_2$

$f_{\ddot{\mathcal{G}}(e)}(y_i)$; $e \in (B-A), \forall y_i \in U_2$

$\max(f_{\ddot{\mathcal{F}}(e)}(x_i), f_{\ddot{\mathcal{G}}(e)}(x_i))$; $e \in (A \cap B), \forall x_i \in U_1$

$\max(f_{\ddot{\mathcal{F}}(e)}(y_i), f_{\ddot{\mathcal{G}}(e)}(y_i))$; $e \in (A \cap B), \forall y_i \in U_2$

$1-f_{\ddot{M}(e)} = 1-t_{\ddot{F}(e)}(x_i)$; $e\in(A-B), \forall x_i \in U_1$

$1-t_{\ddot{G}(e)}(x_i)$; $e\in(B-A), \forall x_i \in U_1$

$1-t_{\ddot{F}(e)}(y_i)$; $e\in(A-B), \forall y_i \in U_2$

$1-t_{\ddot{G}(e)}(y_i)$; $e\in(B-A), \forall y_i \in U_2$

$\max((1-f_{\ddot{F}(e)}(x_i), 1-f_{\ddot{G}(e)}(x_i))$; $e\in(A\cap B), \forall x_i \in U_1$

$\max(1-f_{\ddot{F}(e)}(y_i), 1-f_{\ddot{G}(e)}(y_i))$; $e\in(A\cap B), \forall y_i \in U_2$

Here $C\subseteq N$ and $\forall e\in C$, $\ddot{H}^c(e)=\ddot{M}(e)$

Thus $((\ddot{F}, A)\ddot{\cup}(\ddot{G}, B))^c \subseteq (\ddot{F}, A)^c \cup (\ddot{G}, B)^c$

(2)Proof is similar to (1)

Proposition 5.10:(De- Morgan's law)

For any VBSS $(\ddot{F}, A)$ and $(\ddot{G}, B)$ over absolute VBSS $(\ddot{U}, E)$

(1) $((\ddot{F}, A)\ddot{\cup}(\ddot{G}, A))^c = (\ddot{F}, A)^c \ddot{\cap} (\ddot{G}, B)^c$

(2) $((\ddot{F}, A)\ddot{\cap}(\ddot{G}, B))^c = (\ddot{F}, A)^c \ddot{\cup} (\ddot{G}, B)^c$

Proof

(1) Let $(\ddot{F}, A)\ddot{\cup}(\ddot{G}, A) = (\ddot{H}, A)$

$\forall e\in A$, $\ddot{H}(e)=\ddot{F}(e)\ddot{\cup}\ddot{G}(e)$

$$=\left\{\left(\langle\frac{[t_{\ddot{F}(e)}(x_i), 1-f_{\ddot{F}(e)}(x_i)]}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{[t_{\ddot{F}(e)}(y_i), 1-f_{\ddot{F}(e)}(y_i)]}{y_i}; \forall y_i \in U_2\rangle\right)\right\} \ddot{\cup}$$

$$\left\{\left(\langle\frac{[t_{\ddot{G}(e)}(x_i), 1-f_{\ddot{G}(e)}(x_i)]}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{[t_{\ddot{G}(e)}(y_i), 1-f_{\ddot{G}(e)}(y_i)]}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$$

$$=\left\{\left(\begin{array}{l}\langle\frac{\max\left((t_{\ddot{F}(e)}(x_i), t_{\ddot{G}(e)}(x_i)\right), \max((1-f_{\ddot{F}(e)}(x_i), 1-f_{\ddot{G}(e)}(x_i))}{x_i}; \forall x_i \in U_1\rangle, \\ \langle\frac{\max\left((t_{\ddot{F}(e)}(y_i), t_{\ddot{G}(e)}(y_i)\right), \max((1-f_{\ddot{F}(e)}(y_i), 1-f_{\ddot{G}(e)}(y_i))}{y_i}; \forall y_i \in U_2\rangle\end{array}\right)\right\}$$

$$\ddot{H}(e)^c = \left\{\left(\begin{array}{l}\langle\frac{\min\left((t_{\ddot{F}(e)}(x_i), t_{\ddot{G}(e)}(x_i)\right), \min((1-f_{\ddot{F}(e)}(x_i), 1-f_{\ddot{G}(e)}(x_i))}{x_i}; \forall x_i \in U_1\rangle, \\ \langle\frac{\min\left((t_{\ddot{F}(e)}(y_i), t_{\ddot{G}(e)}(y_i)\right), \min((1-f_{\ddot{F}(e)}(y_i), 1-f_{\ddot{G}(e)}(y_i))}{y_i}; \forall y_i \in U_2\rangle\end{array}\right)\right\}, \forall e\in A$$

$$\Rightarrow(\ddot{H}, A) = \left\{\left(\begin{array}{l}\langle\frac{\min\left(\left(t_{\ddot{F}(e)}(x_i), t_{\ddot{G}(e)}(x_i)\right), \min((1-f_{\ddot{F}(e)}(x_i), 1-f_{\ddot{G}(e)}(x_i)\right)}{x_i}; \forall x_i \in U_1\rangle, \\ \langle\frac{\min\left(t_{\ddot{F}(e)}(y_i), t_{\ddot{G}(e)}(y_i)\right), \min((1-f_{\ddot{F}(e)}(y_i), 1-f_{\ddot{G}(e)}(y_i))}{y_i}; \forall y_i \in U_2\rangle\end{array}\right)\right\}$$

Let $(\ddot{F}, A)^c \ddot{\cap} (\ddot{G}, B)^c = (\ddot{M}, A)$

$\forall$ e∈A, $\ddot{M}(e) = \ddot{F}(e)^c \ddot{\cup} \ddot{G}(e)^c$

$$= \left\{\left(\langle\frac{[f_{\ddot{F}(e)}(x_i), 1-t_{\ddot{F}(e)}(x_i)]}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{[f_{\ddot{F}(e)}(y_i), 1-t_{\ddot{F}(e)}(y_i)]}{y_i}; \forall y_i \in U_2\rangle\right)\right\} \ddot{\cap}$$

$$\left\{\left(\langle\frac{[f_{\ddot{G}(e)}(x_i), 1-t_{\ddot{G}(e)}(x_i)]}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{[f_{\ddot{G}(e)}(y_i), 1-t_{\ddot{G}(e)}(y_i)]}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$$

$$= \left\{\left(\begin{array}{l}\langle\frac{\min\left(\left(t_{\ddot{F}(e)}(x_i), t_{\ddot{G}(e)}(x_i)\right), \min((1-f_{\ddot{F}(e)}(x_i), 1-f_{\ddot{G}(e)}(x_i)\right)}{xi}; \forall x_i \in U_1\rangle, \\ \langle\frac{\min\left(t_{\ddot{F}(e)}(y_i), t_{\ddot{G}(e)}(y_i)\right), \min((1-f_{\ddot{F}(e)}(y_i), 1-f_{\ddot{G}(e)}(y_i))}{y_i}; \forall y_i \in U_2\rangle\end{array}\right)\right\}$$

$\Rightarrow(\ddot{M}, A) = (\ddot{F}, A)^c \ddot{\cap} (\ddot{G}, B)^c \Rightarrow (\ddot{H}, A) = (\ddot{M}, A)$.

(2) Proof is similar to (1)

Proposition 5.11:(Idempotent property for AND & OR operation)

For any vague binary soft set $(\ddot{F}, A)$ over the absolute vague binary soft set $(\ddot{U}, E)$

(1) $(\ddot{F}, A)\ddot{\wedge}(\ddot{F}, A) = (\ddot{F}, A)$

(2) $(\ddot{F}, A)\ddot{\vee}(\ddot{F}, A) = (\ddot{F}, A)$

Proof

$(\ddot{F}, A)\ddot{\wedge}(\ddot{F}, A) = (\ddot{M}, C)$ where C= A×A, ∀(a, a)∈ C=(A×A),

$\ddot{M}(a,a) = \ddot{F}(a)\ddot{\cap}\ddot{F}(a) = \ddot{F}(a) \Rightarrow (\ddot{M}, C) = (\ddot{F}, A) \Rightarrow (\ddot{F}, A)\ddot{\wedge}(\ddot{F}, A) = (\ddot{F}, A)$

(2) Proof is similar to (1)

Proposition 5.12:

(1) $(\ddot{F}, A)\ddot{\cup}(\ddot{\emptyset}, B) = (\ddot{F}, A) \Leftrightarrow B \subseteq A$

(2) $(\ddot{F}, A)\ddot{\cup}(\ddot{U}, B) = (\ddot{U}, A) \Leftrightarrow A \subseteq B$

Proof

$(\ddot{F}, A) = \{e \in A / (e, \ddot{F}(e))\}$ where $\ddot{F}(e) = \left\{\left(\langle\frac{V_{\ddot{F}(e)}(x_i)}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{V_{\ddot{F}(e)}(y_i)}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$

Also let $(\ddot{\emptyset}, B) = (\ddot{G}, B)$

$\forall e \in B$, $\ddot{G}(e) = \left\{\left(\langle\frac{[0,0]}{x_i}; \forall x_i \in U_1\rangle, \langle\frac{[0,0]}{y_i}; \forall y_i \in U_2\rangle\right)\right\}$

Let$(\ddot{\mathcal{F}}, A)\ddot{\cup}(\ddot{\mathcal{G}}, B) = (\ddot{\mathcal{N}}, C)$ where C=AUB

$\forall e\in C$, $\ddot{\mathcal{N}}(e) = \ddot{\mathcal{F}}(e)$; $e\in (A-B)$

$\ddot{\mathcal{G}}(e)$; $e\in (B-A)$

$\ddot{\mathcal{F}}(e)\ddot{\cup}\ddot{\mathcal{G}}(e)$; $e\in (A\cap B)$

$$= \left\{\left(\langle\frac{V_{\mathcal{F}(e)}(x_i)}{x_i};\forall x_i \in U_1\rangle,\langle\frac{V_{\mathcal{F}(e)}(y_i)}{y_i};\forall y_i \in U_2\rangle\right)\right\};\ e\in (A-B)$$

$$\left\{\left(\langle\frac{[0,0]}{x_i};\forall x_i \in U_1\rangle,\langle\frac{[0,0]}{y_i};\forall y_i \in U_2\rangle\right)\right\};\ e\in (B-A)$$

$$\left\{\left(\langle\frac{\max(V_{\mathcal{F}(e)}(x_i),[0,0])}{x_i};\ \forall x_i \in U_1\rangle,\langle\frac{\max(V_{\mathcal{F}(e)}(y_i),[0,0]}{y_i};\ \forall y_i \in U_2\rangle\right)\right\};\ e\in (A\cap B)$$

$$= \left\{\left(\langle\frac{V_{\mathcal{F}(e)}(x_i)}{x_i};\forall x_i \in U_1\rangle,\langle\frac{V_{\mathcal{F}(e)}(y_i)}{y_i};\forall y_i \in U_2\rangle\right)\right\};\ e\in (A-B)$$

$$\left\{\left(\langle\frac{[0,0]}{x_i};\forall x_i \in U_1\rangle,\langle\frac{[0,0]}{y_i};\forall y_i \in U_2\rangle\right)\right\}\qquad ;\ e\in (B-A)$$

$$\left\{\left(\langle\frac{V_{\mathcal{F}(e)}(x_i)}{x_i};\ \forall x_i \in U_1\rangle,\langle\frac{V_{\mathcal{F}(e)}(y_i)}{y_i};\forall y_i \in U_2\rangle\right)\right\};\ e\in (A\cap B)$$

Let $B\subseteq A$,

$$\ddot{\mathcal{N}}(e)= \left\{\left(\langle\frac{V_{\mathcal{F}(e)}(x_i)}{x_i};\forall x_i \in U_1\rangle,\langle\frac{V_{\mathcal{F}(e)}(y_i)}{y_i};\forall y_i \in U_2\rangle\right)\right\};\ e\in (A-B)$$

$$\left\{\left(\langle\frac{V_{\mathcal{F}(e)}(x_i)}{x_i};\forall x_i \in U_1\rangle,\langle\frac{V_{\mathcal{F}(e)}(y_i)}{y_i};\forall y_i \in U_2\rangle\right)\right\};\ e\in (A\cap B)$$

$= \ddot{\mathcal{F}}(e)$, $\forall e\in A$

Conversely $(\ddot{\mathcal{F}}, A)\ddot{\cup}(\ddot{\emptyset}, B) = (\ddot{\mathcal{F}}, A) \Rightarrow AUB = A \Rightarrow B\subseteq A$

(2) Proof is similar to (1)

7. Conclusion

Vague binary soft sets are introduced with two initial universes. It is hoped that this hybrid form can extract positive qualities of both vague and soft sets and would be more useful while handling real life situations with uncertainties and vagueness.

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